Vibration Analysis

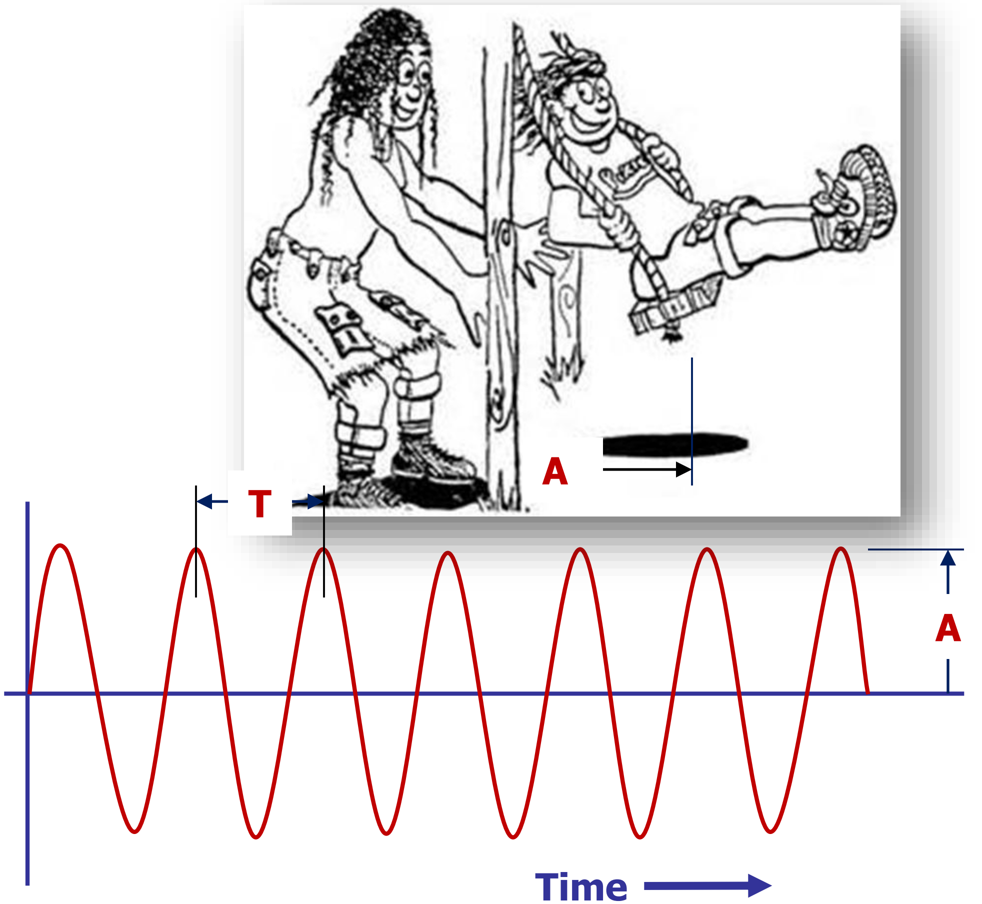
BASICS

Presented by: Ray Delaforce

Basic concepts

Any physical vibrating system in motion is subject to Newton's laws

A vibrating object has a maximum displacement called the amplitude A



Basic concepts

Any physical vibrating system in motion is subject to Newton's laws

A vibrating object has a maximum displacement called the amplitude A

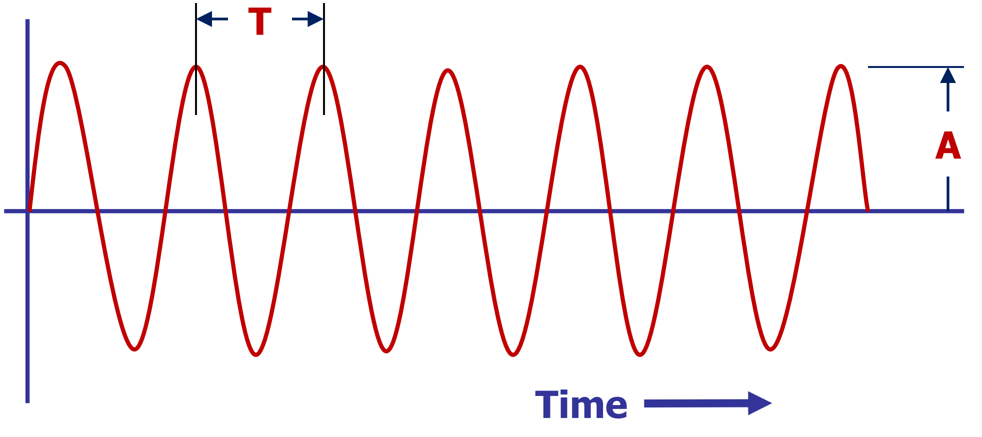
There is also a period (time) from crest to crest T

The vibrating object also must have mass M

The elements have to satisfy Newton's Second Law of motion

What is needed is the force F to act on this mass

A device that produces a cyclic force is a spring



Basic concepts

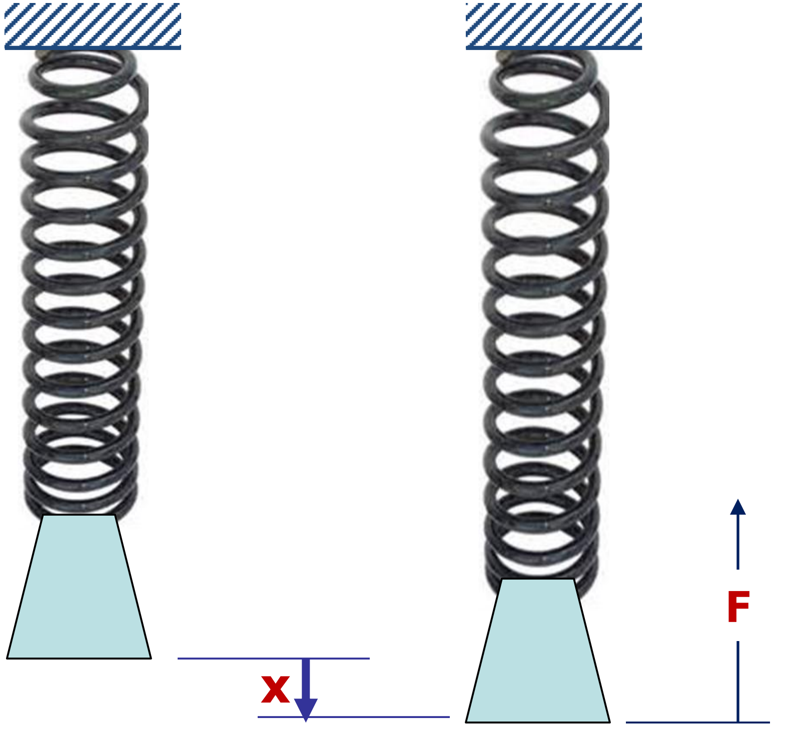
Consider first **Newton's Second Law** of motion

Force = Mass x Acceleration, or $F = M.a$

Here is a mass on a spring , that has been displaced a distance x

There is a force in the upward (-ve) direction on the mass $-F$

If k is the spring stiffness, the force is $F = -k.x$



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The acceleration is the second derivative of x, thus $a = \ddot{x}$

From Newton's law we can get the differential equation

$$F = M.a$$

$$-k.x \quad = \quad \text{Mass} \times \ddot{x} \text{ (acceleration)}$$

In the more familiar form:

$$M.\ddot{x} + k.x = 0$$

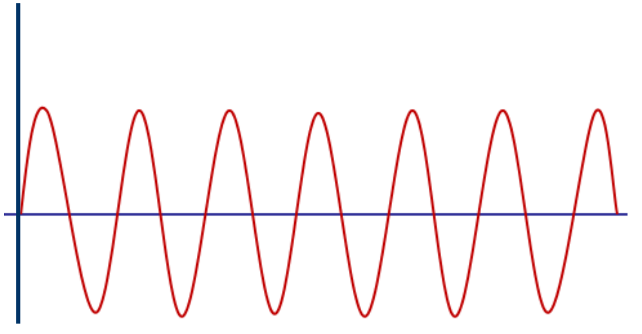
$$\ddot{x} = \frac{d^2x}{dt^2}$$

Basic concepts

$$M.\ddot{x} + k.x = 0$$

The solution to this equation for the **displacement x** is:

$$x = A.\sin\left(\sqrt{\frac{k}{M}}.t\right)$$



If you would like the derivation:

$$\ddot{x} + \frac{k}{M}x = 0 \qquad \text{Let } x = Ae^{\alpha t} \qquad \alpha^2 Ae^{\alpha t} + Ae^{\alpha t}\frac{k}{M} = 0 \qquad \alpha^2 + \frac{k}{M} = 0$$

$$\alpha = i\sqrt{\frac{k}{M}} \qquad x = Ae^{i\sqrt{\frac{k}{M}}.t}$$

Thus: $x = A.\sin\left(\sqrt{\frac{k}{M}}.t\right) + i.A.\cos\left(\sqrt{\frac{k}{M}}.t\right)$

But x does not contain a imaginary part thus: $x = A.\sin\left(\sqrt{\frac{k}{M}}.t\right)$

Basic concepts

$$M.\ddot{x} + k.x = 0$$

The solution to this equation for the displacement x is:

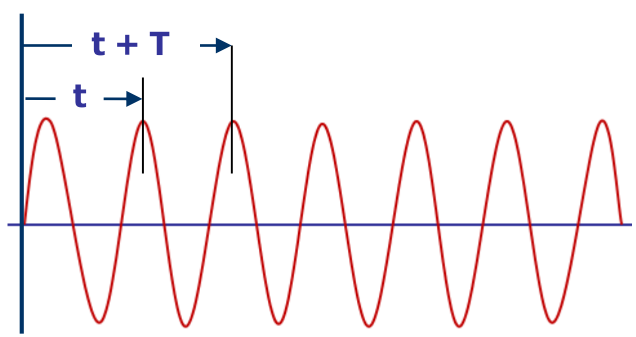
$$x = A.\sin\left(\sqrt{\frac{k}{M}}.t\right)$$

We can also find the period of vibration T :

$$T = 2\pi\sqrt{\frac{M}{k}}$$

If you would like the derivation:

$$A.\sin\left[\sqrt{\frac{k}{M}}.(t + T)\right] = A.\sin\left[\sqrt{\frac{k}{M}}.t + 2\pi\right]$$



Basic concepts

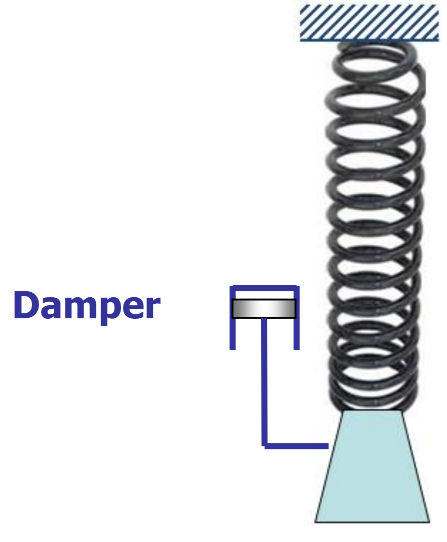
Looking at the sine wave, is seems to go on forever

Cannot happen in a real physical system, it must diminish over time

We can represent this reduction of amplitude with a damper

This modifies the equation to become:

$$M.\ddot{x} + c.\dot{x} + k.x = 0$$



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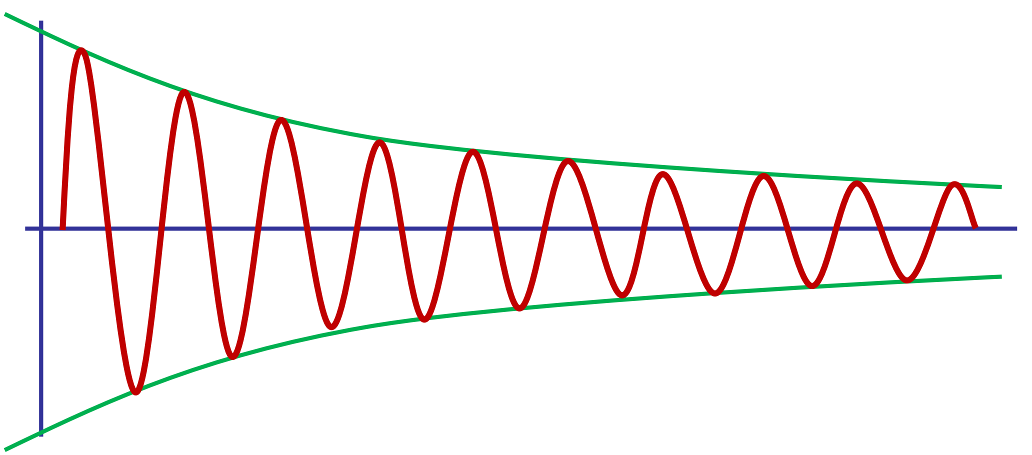
Where c can be taken as the damping factor:

The damped vibration look like this, this is the damping envelope



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Where c can be taken as the damping factor:

The damped vibration look like this, this is the damping envelope

This assumes the system experiences just an impact load once

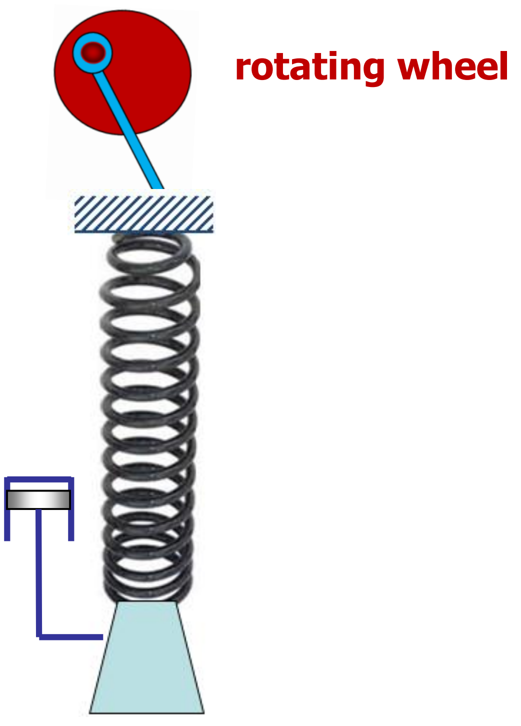
If the system is subjected to continuous force the equation becomes:

$$M.\ddot{x} + c.\dot{x} + k.x = \text{force input}$$

The solution takes long to derive, even for simple force input

Basic concepts

Our simple spring model now looks like this, with force input



This simple model helps to explain what happens with a vessel during a seismic event, which is a much more complicated system

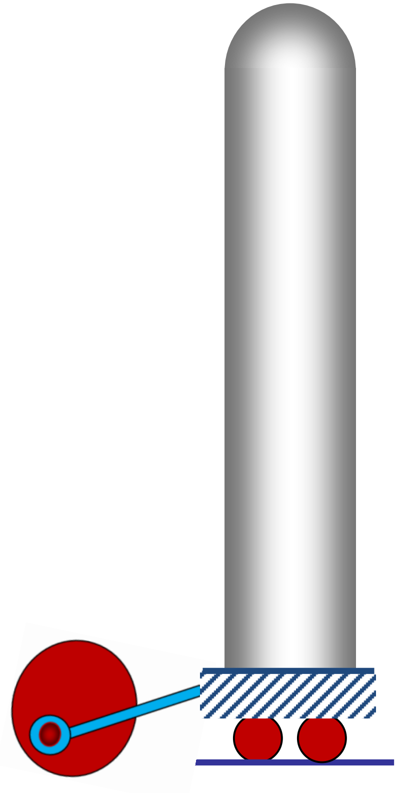
Basic concepts

A vertical vessel stands on its foundation like this

As the ground shakes during a seismic event, this is what happens

The vessel is shaken from side to side, producing stresses in the shell

The vessel wants to do this



Basic concepts

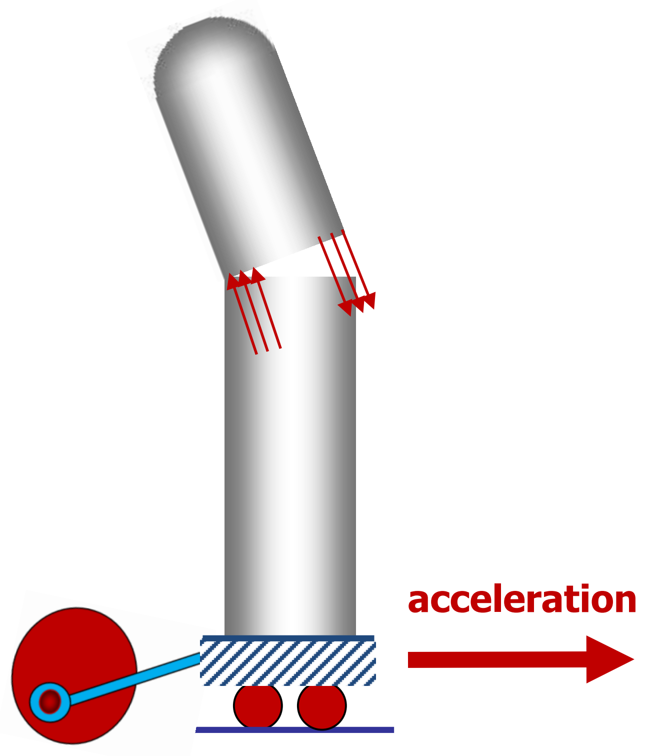
A vertical vessel stands on its foundation like this

As the ground shakes during a seismic event, this is what happens

The vessel is shaken from side to side, producing stresses in the shell

The vessel wants to do this, as the vessel is accelerated right

Produces a tensile stress in the shell wall, compressive the other side



Basic concepts

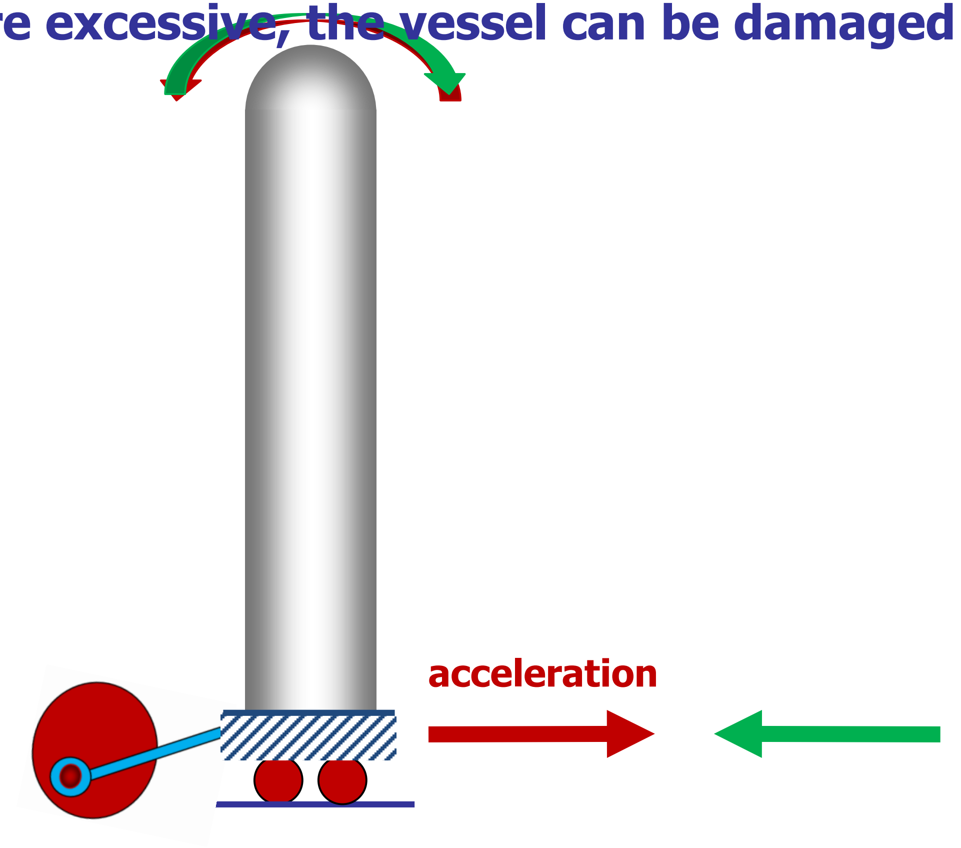
A vertical vessel stands on its foundation like this

As the ground shakes during a seismic event, this is what happens

This is the same as subjecting the vessel to a moment

Vibration takes place both to the left and to the right , reversing moment

If the oscillations are excessive, the vessel can be damaged



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If the oscillations are excessive, the vessel can be damaged

If the natural frequency coincides with the forcing frequency – damage can be the result !

Basic concepts

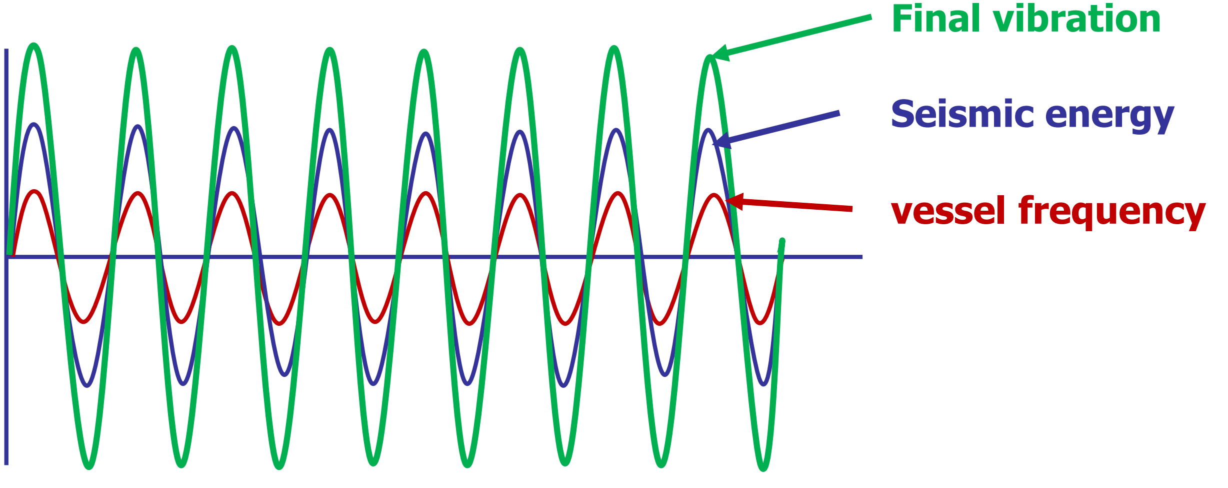
This is the natural frequency of the vessel

This is the forcing frequency from the seismic event, input energy

The two frequencies are added together, called resonance

That vibration can damage attached piping and the foundation & bolting

The forcing seismic energy is not a simple sine wave



Basic concepts

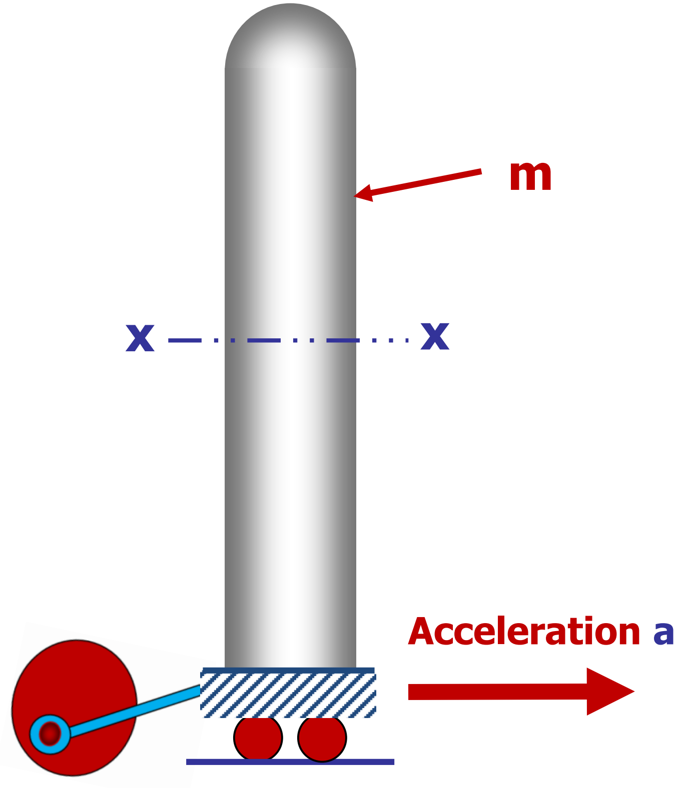
Before we look at seismic profiles, let us look at this simple case

A vessel experiencing an acceleration a (m/s^2) to the right

The part of the vessel above section x-x has a mass of m (kg)

From Newton's Second Law there is a shear force f = ma

This acts at the Centre of Gravity of the section



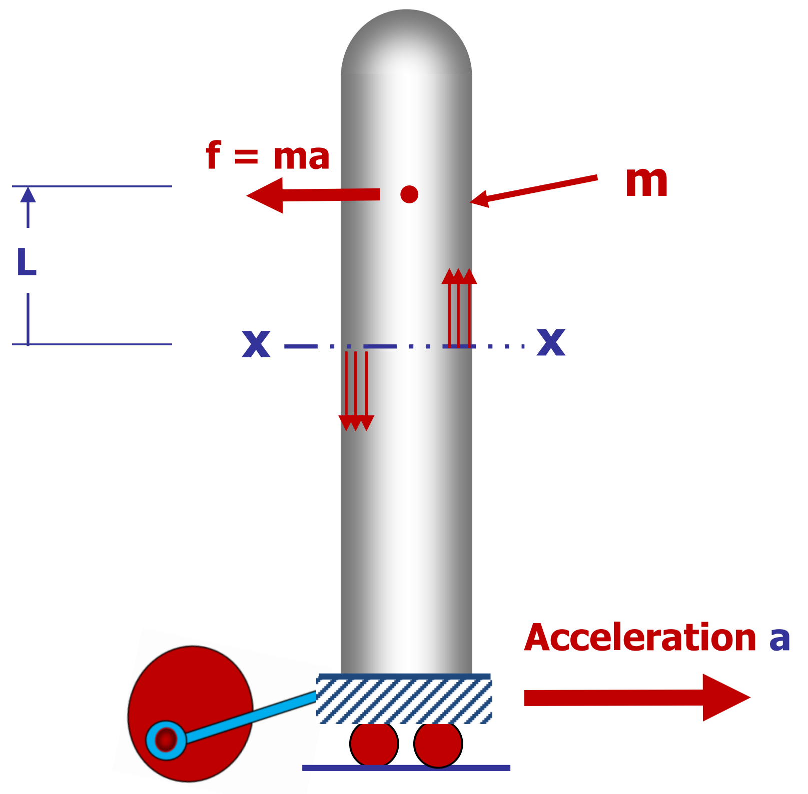
Basic concepts

This is the force that acts at the CG of the upper section

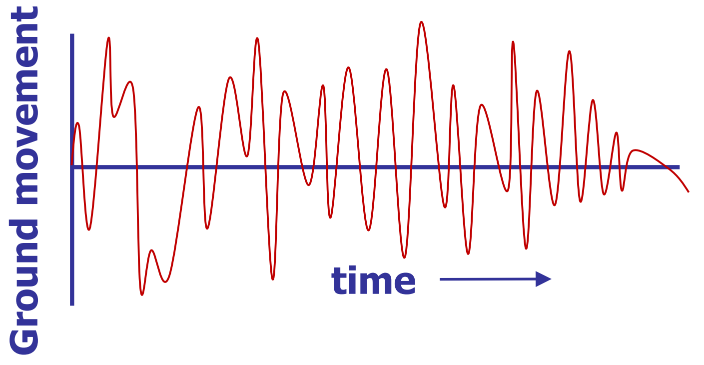
This causes a moment about x-x , M = f.L

Giving rise to the stresses in the lower portion of the vessel shell

Things are not that simple !



The real profile of a Seismic Event



This data could come from a real Earthquake in California for example

How do we deal with such complicated profiles ?

The four things that are needed are:

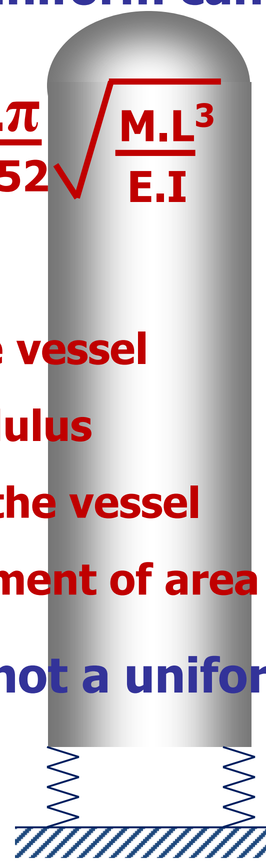
- The **period** of vibration
- The **mode** of vibration
- The properties of the **soil/rock** on which the vessel stands
- The type of seismic event

The soil or rock has partially elastic properties

It’s as though the foundation were springs under the vessel

There are several ways of estimating the period of vibration

If the vessel were a simple uniform cantilever the period is:



$$T = \frac{2.\pi}{3,52}\sqrt{\frac{M.L^3}{E.I}}$$

Were:

M = Mass of the vessel

E = Elastic Modulus

L = Length of the vessel

I = Second Moment of area of the vessel

But, invariably the vessel is not a uniform structure

C. E. Freese wrote a paper to estimate the period of vibration

Vibrations of Vertical Pressure Vessels

C. E. FREESE

Mechanical Engineer, The Fluor Corporation, Ltd., Los Angeles, Calif. Mem. ASME

This paper is primarily concerned with the vibration of vertical pressure vessels known as columns or towers.

The procedure for estimating the period of first mode of vibration for columns which are the same diameter and thickness for their entire length is outlined. A graph is included for this purpose which recommends limits between vessels considered to be static structures and those considered dynamic.

A method for designing vessels considered as dynamic structures is described as well as a detailed procedure for estimating the period of vibration of multithickness (stepped shell) vessels and/or vessels built to two or more diameters with conical transitions where the difference in diameter is small.

There is a brief resume of the "Karman vortexes" effect and a discussion regarding vibration damping by liquid loading and the benefit of ladders and platforms which help reduce the effect of periodic eddy shedding.

The design procedure outlined will be useful to the practical vessel designer confronted with the task of investigating vibration possibilities in vertical pressure vessels.

The drawback:

- It only estimates the first mode of vibration
- It is an estimate; accuracy depending the segment lengths

C.E. Freese tried to estimate if the vibrations were not too wild

Introduction

For many years it was customary to apply guy wires to tall ,slender pressure vessels. In, recent years, refinery and petro-chemical officials have demanded self-supporting vessels from the standpoint of plant appearance and safety.

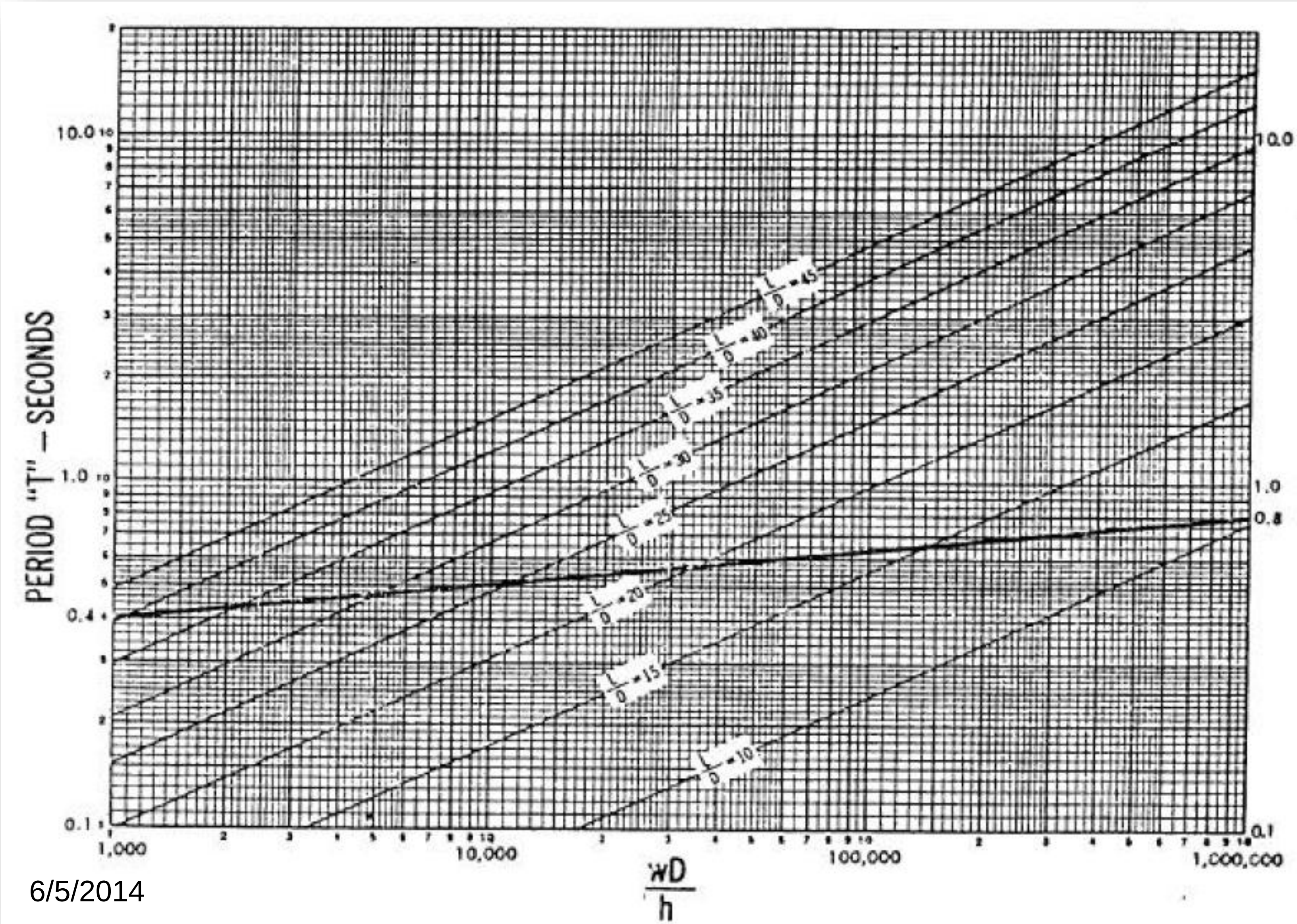
In order to design a self-supporting vessel of this type, the following problems must be carefully analyzed:

2 What is the most practical method for designing to meet dynamic conditions?

3 Does the method used produce consistent results and does it provide additional strength to resist the force due to the mass-acceleration resulting from the motion of the vessel ?

4 Is the period of vibration of the dynamically designed vessel such that prevailing winds are not apt to cause excessive movement?

C. E. Freese used this graph to estimate the stability



Look at these Modes of vibration

The simple numerical integration cannot handle higher modes

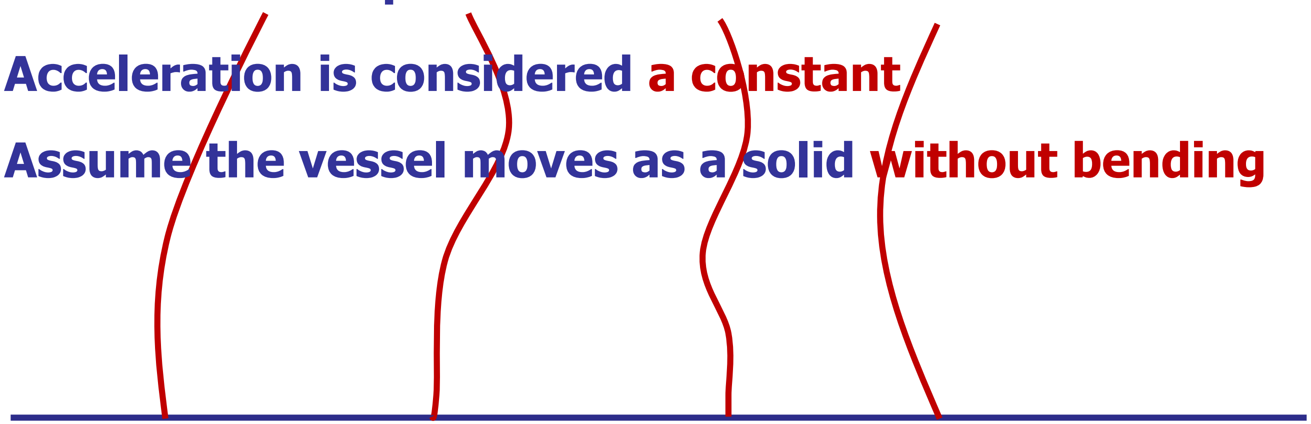
For accurate work another method must be employed

PV Elite uses an **Eigen Solver** to accurately predict the period of many modes of vibration

But first, we consider a very simple analysis, assuming we know the **g acceleration** of the vessel

These are the assumptions:

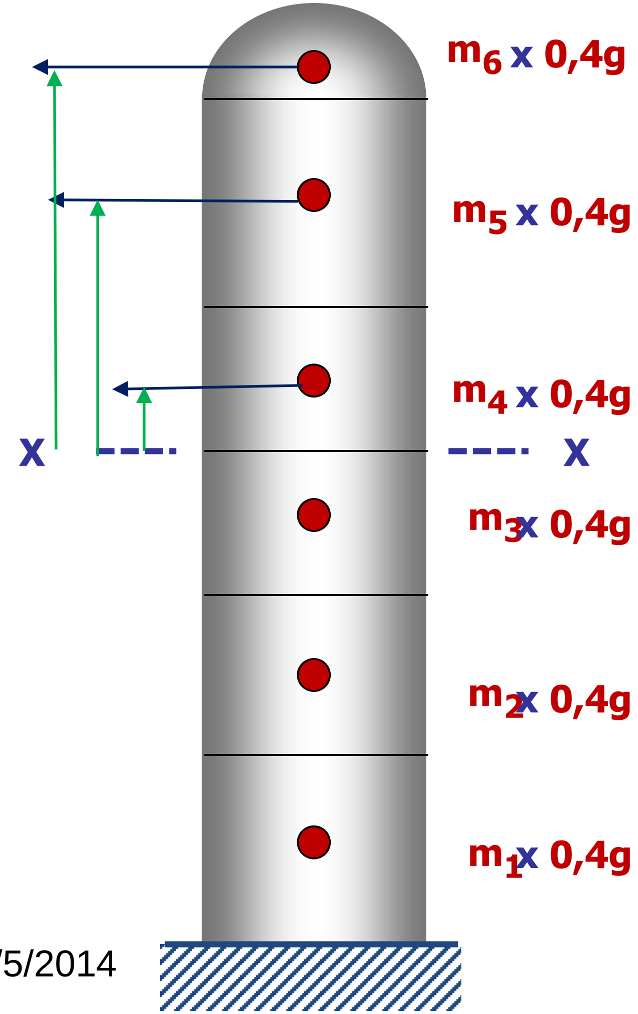
- Acceleration is considered **a constant**
- Assume the vessel moves as a solid **without bending**



The simple approach (very conservative)

First set up the model, divided into shell sections, with the CG's

Get the mass of each section, set the acceleration 0,4g (say)



Compute the inertial force of each section

Choose a section

Forces at the centroids

Set the moment arms

Moment = Σ(force x moment arm)

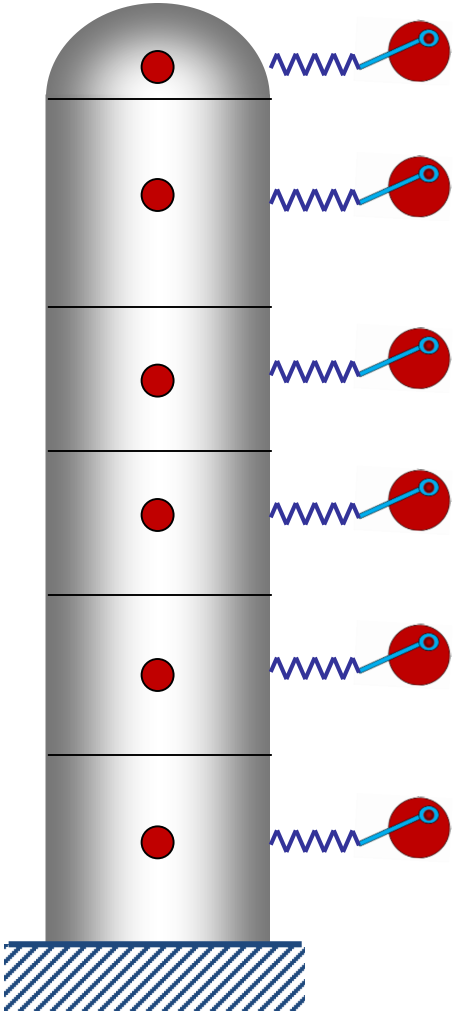
The tensile and compressive stresses can now be computed at section X-X

Example Seismic Code ASCE 7: 2002

In an earthquake the system of dynamic forces is very complex

As though each mass is excited by a number of oscillators

Presents a complex dynamic calculation to compute vibration period



PV Elite has an Eigen solver which

- Computes the period of vibration accurately
- Does the calculation for many modes
- Is less conservative than the numerical method
- Reduces stresses

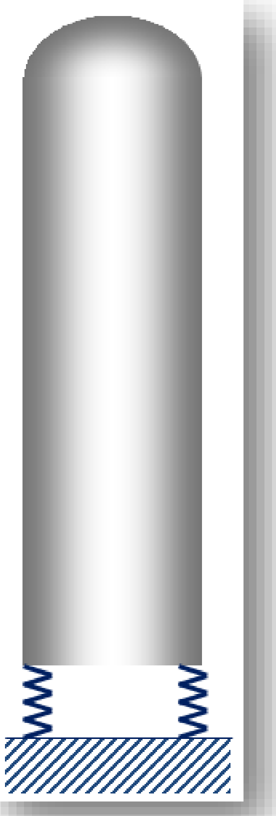
Demo ***Vibration_Seismic.pvdb***

Example Seismic Code ASCE 7: 2002

First we have to classify the site where the vessel is located

There are five site classes: A, B, C, D, E and F

- **A hard rock** v_s > 1500 m/s
- **B Rock** 760 m/s < v_s <1500 m/s
- **C dense soil** 370 m/s < v_s < 760 m/s
- **D stiff soil** 180 m/s < v_s < 370 m/s
- **E soil** v_s < 180 m/s
- **F sites requiring an engineering survey**



The site class defines the elastic quality of the site

Sites where the soil can be liquefied need special consideration.

The site class is generally applied to the upper layers of the ground

Example Seismic Code ASCE 7: 2002

Assign the short period spectral acceleration S_S and S_1

S_S = acceleration for 5% damping at short period

S_1 = acceleration for 5% damping at 1 second period

Here are values from the United States Geological Survey

50 U.S. States

Name	Min S_S	Min S_1	Max S_S	Max S_1
Alabama	0.083g (30.200°N, 87.650°W)	0.051g (30.180°N, 87.800°W)	0.401g (35.000°N, 88.200°W)	0.175g (35.000°N, 88.200°W)
Alaska	0.006g (71.400°N, 156.600°W)	0.003g (71.200°N, 156.200°W)	2.865g (59.000°N, 137.900°W)	2.482g (59.000°N, 137.900°W)
Arizona	0.127g (36.790°N, 109.050°W)	0.045g (36.500°N, 109.200°W)	0.978g (32.500°N, 114.810°W)	0.333g (32.500°N, 114.810°W)
Arkansas	0.135g (33.020°N, 94.040°W)	0.074g (33.020°N, 94.040°W)	2.814g (36.000°N, 89.800°W)	1.117g (36.000°N, 89.800°W)
California	0.204g (34.350°N, 114.180°W)	0.107g (34.350°N, 114.180°W)	3.730g (34.460°N, 119.010°W)	1.389g (34.400°N, 118.760°W)
Colorado	0.070g (40.700°N, 102.060°W)	0.036g (40.700°N, 102.060°W)	0.477g (37.150°N, 105.500°W)	0.143g (37.700°N, 105.550°W)
Connecticut	0.155g (41.310°N, 71.910°W)	0.057g (41.310°N, 71.910°W)	0.264g (41.000°N, 73.660°W)	0.071g (41.100°N, 73.720°W)

Example Seismic Code ASCE 7: 2002

Assign the short period spectral acceleration S_S and S_1

Next we need some coefficients F_a and F_v, Tables 9.4.1.2.4 a and b

TABLE 9.4.1.2.4a

VALUES OF F_a AS A FUNCTION OF SITE CLASS AND MAPPED SHORT PERIOD MAXIMIMUM CONSIDERED EARTHQUAKE SPECTRAL ACCELERATION

	Mapped Maximum Considered Earthquake Spectral Response Acceleration at Short Period				
Site Class	S_S<0.25	S_S=0.5	S_S=0.75	S_S=1.0	S_S=1.25
A	0.8	0.8	0.8	0.8	0.8
B	1.0	1.0	1.0	1.0	1.0
C	1.2	1.2	1.1	1.0	1.0
D	1.6	1.4	1.2	1.1	1.0
E					
F					

TABLE 9.4.1.2.4b

VALUES OF F_V AS A FUNCTION OF SITE CLASS AND MAPPED 1-SECOND PERIOD MAXIMUM CONSIDERED EARTHQUAKE SPECTRAL ACCELERATION

	Mapped Maximum Considered Earthquake Spectral Response Acceleration at 1-Second Periods				
Site Class	S_1<0.1	S_1=0.2	S_1=0.3	S_1=0.4	S_1=0.5
A	0.8	0.8	0.8	0.8	0.8
B	1.0	1.0	1.0	1.0	1.0
C	17	1.6	1.5	1.4	1.3
D	2.4	2.0	1.8	1.6	1.5
E	3.5	3.2	2.8	2.4	2.4
F	a	a	a	a	a

Example Seismic Code ASCE 7: 2002

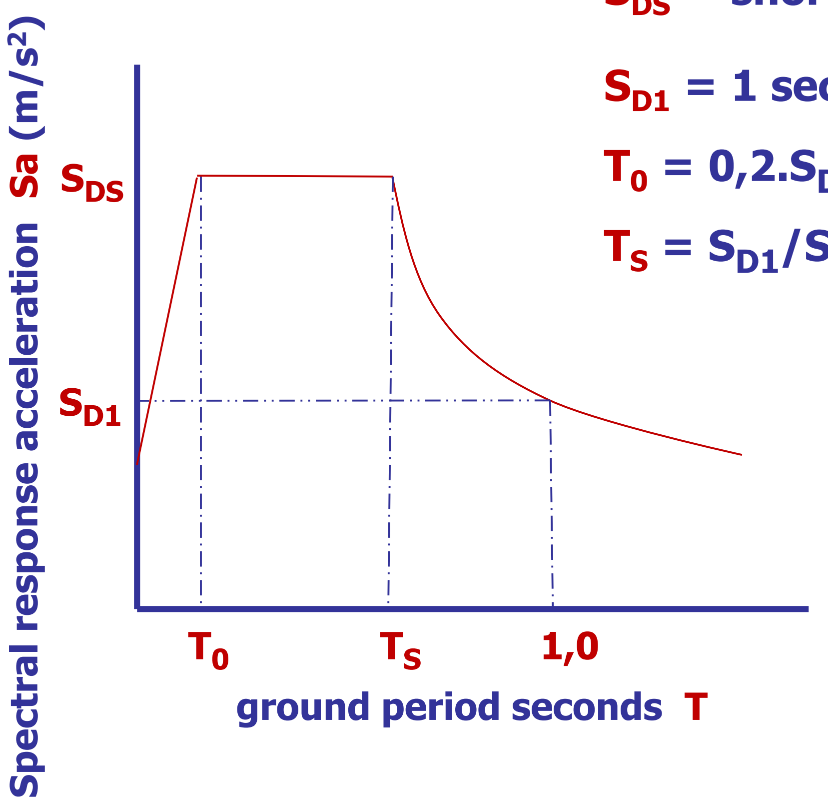
Next we have to find the Site Adjustment Coefficient

$$S_{MS} = F_a.S_S \qquad S_{M1} = F_V.S_1$$

Now find the Design Spectral Response Acceleration

$$S_{DS} = \frac{2}{3} S_{MS} \qquad S_{D1} = \frac{2}{3} S_{M1}$$

Here is the information as seen on the response spectrum diagram



S_{DS} = short time response

S_{D1} = 1 second response

$T_0 = 0{,}2.S_{D1}/S_{DS}$

$T_S = S_{D1}/S_{DS}$

Example Seismic Code ASCE 7: 2002

Next the Coefficient for Upper Calculated Period C_U

TABLE 9.5.5.3.1
COEFFICIENT FOR UPPER LIMIT ON
CALCULATED PERIOD

Design Spectral Response Acceleration at 1 Second S_{D1}	Coefficient C_U
> 0.4	1.4
0.3	1.4
0.2	1.5
0.15	1.6
0.1	1.7
< 0.05	1.7

We can also get Approximate Period Parameters C_t and x

TABLE 9.5.5.3.2
VALUES OF APPROXIMATE PERIOD PARAMETERS C_t AND x

Structure Type	C_t	x
Moment resisting frame systems of steel in which the frames resist 100% of the required seismic force and are not enclosed or adjoined by more rigid components that will prevent the frames from deflecting when subjected to seismic forces	0.028(0.068)	0.8
Moment resisting frame systems of reinforced concrete in which the frames resist 100% of the required seismic force and are not enclosed or adjoined by more rigid components that will prevent the frames from deflecting when subjected to seismic forces	0.028(0.044)	0.9
Eccentrically braced steel frames	0.03(0.07)	0.75
All other structural systems	0.02(0.055)	0.75

Example Seismic Code ASCE 7: 2002

Now we can compute the fundamental Period of Vibration T_a

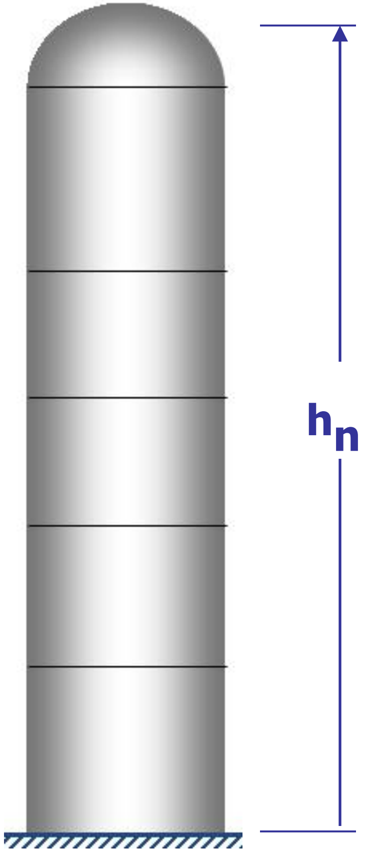
$$T_a = C_t \cdot (h_n)^x$$

Where h_n is the height of the vessel

The actual fundamental frequency of Vibration of the vessel is T

We limit the maximum frequency in the analysis:

$$T = \min(T;\ C_U . T_a)$$



Example Seismic Code ASCE 7: 2002

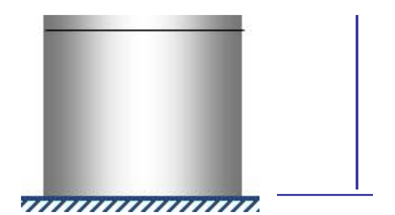
We need R, Response Modification Coefficient

TABEL 9.5.2.2
DESIGN COEFFICIENT AND FACTORS FOR BASIC SEISMIC FORCE–RESISTING SYSTEMS

Basic Seismic for Resisting-system	Response Modification Coefficient, R	System Over-strength Factor W_o	Deflection Amplification Factor C_d	Structural System Limitations & Building Height (ft) Limitations				
				Seismic Design Category				
				A&B	C	D	E	F
Inverted Pendulum System and Cantilevered Column Systems								
Special steel moment frames	2.5	2	2.5	NL	NL	NL	NL	NL
Ordinary steel moment frames	1.25	2	2.5	NL	NL	NP	NP	NP
Special reinforced concrete moment frames	2.5	2	1.25	NL	NL	NP	NP	NP
Structural Steel Systems Not Specifically Detailed for Seismic Resistance	**3**	**3**	**3**	NL	NL	NP	NP	NP

Response modification coefficient, R, for use throughout the standard. Note R reduces forces to a strength level, not allowable stress

The best choice is R = 3



Example Seismic Code ASCE 7: 2002

We need I, the Importance Factor from the Seismic Use Group

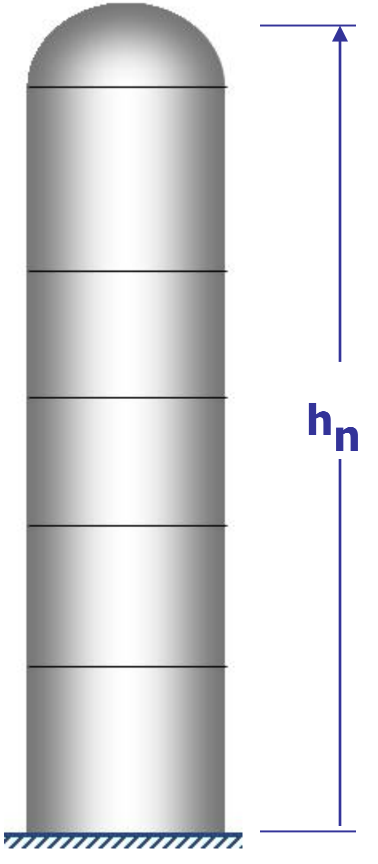
TABLE 9.1.4
OCCUPANCY IMPORTANCE FACTORS

Seismic Use Group	I
I	1
II	1,25
III	1,5

The **SUG** depends on the type of structure

- ❑ High hazard exposure structures
- ❑ Protected access
- ❑ Secure structures

ASCE 7 has all the details



Example Seismic Code ASCE 7: 2002

Next, we need the Seismic Response Coefficient C_S

There are several sets of calculations to determine C_S

We pick the first formula and let it go at that

$$C_S = \frac{S_{DS}}{R/I}$$

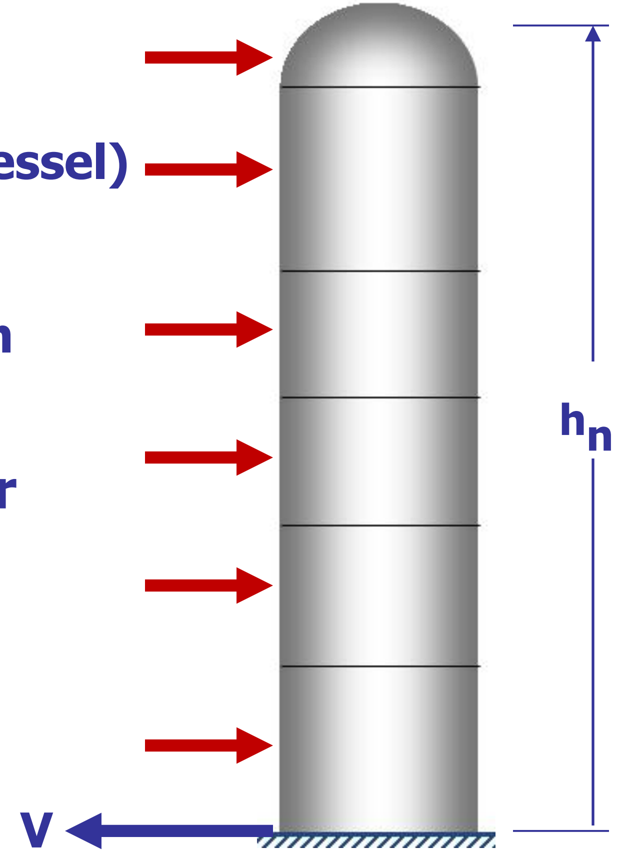
Base shear force $V = C_S.W$ (W = weight of vessel)

Each element of the vessel defines its distribution as it has a particular height from grade

We nee the Vertical Distribution factor k for the vessel, depends on the Period of Vibration

- **If T = 0,5 k = 1**
- **If T = 2,5 k = 2**

Interpolate for intermediate values of T



Example Seismic Code ASCE 7: 2002

We need the weight w_i of each component of the vessel

We now find the Vertical Distribution of the Seismic Forces C_{vx}

$$C_{vx} = \frac{w_x . h_x}{\sum_{i=1}^{n} w_i . h_i}$$

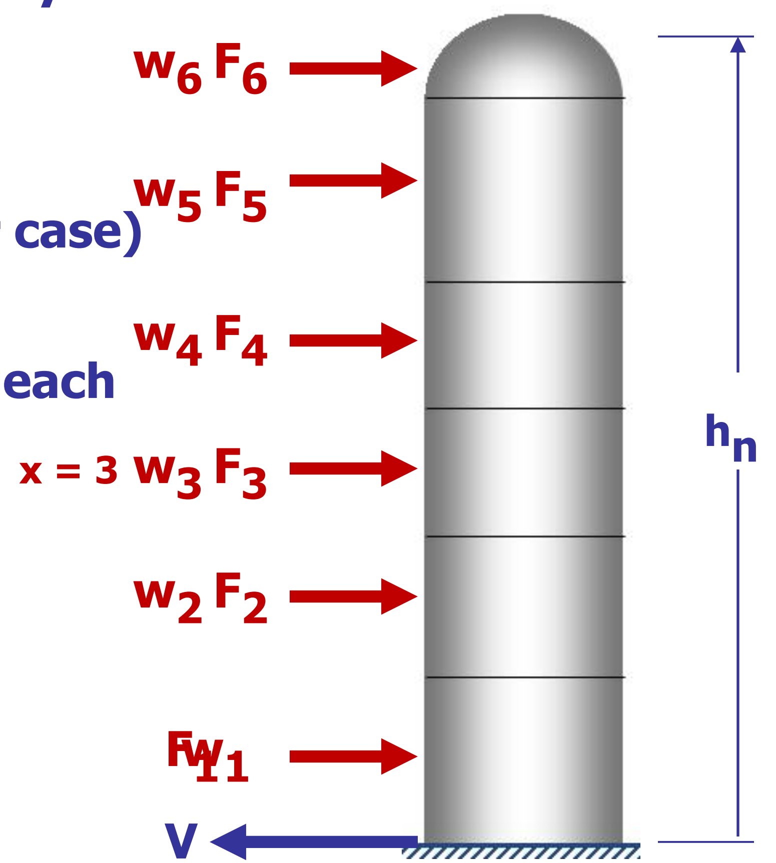
C_{vx} is an array of numbers

Where:

- **n is the number of elements (6 in our case)**
- **x is the element of interest**

Now we can find the Seismic Force on each element F_x

$$F_x = C_{vx} . V$$



Example Seismic Code ASCE 7: 2002

Now we can get the seismic moment at (say) section X-X

From the moments, we can compute the axial stresses